

Optimizing POCS Parameters Through Genetic Programming and its Application for CBERS-2 Satellite Image Restoration

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Abstract. We introduced here a Genetic Programming (GP) approach for optimal or quasi-optimal parameter selection for projection-based image restoration algorithms, such that the well known Projections Onto Convex Sets (POCS) technique for iterative image restoration. We boosted here the RAP-2D convex projector operator by selecting the relaxation parameter that best fit a GP fitness function given by the ISNR index. Experiments in phantom and real CBERS-2 satellite images showed the good results obtained.

Keywords: projections onto convex sets, image restoration, genetic programming projeções em conjuntos convexos, restauração de imagens, programação genética.

1. Introduction

The amount of satellite imagery has widely increased with the new sophisticated onboard imaging systems. Among this new generation of satellites, the CBERS project (China-Brazil Earth Resources Satellite) was jointly developed by Brazil and China and its mission was designed to create four satellites to capture high-resolution images of the Earth by using panchromatic and multispectral detectors. However, remote sensing images still have limitations. Therefore, the images need to be processed to better reflect its radiometric and geometric quality. One of the radiometric correction techniques is image restoration. Its goal is the reconstruction or recovery of the degraded image using *a priori* knowledge of the degradation phenomenon. The optical defocussing, combined with the analog to digital converter and any corrections necessary to compensate for internal detector problems results in an image where the effective instantaneous field of view (EIFOV) is different from the nominal resolution value, resulting in a blurring effect.

The blurring effect, however, can be compensated by a restoration filter that is applied in the image to reduce its blurred appearance. Restoration of multispectral images is crucial because the majority of methods tend to increase the noisiness of the data, thus lowering its radiometric quality. A large number of image restoration methods have been developed for several applications, like Inverse and Wiener Filter, regularization techniques and MAP techniques. However, one of the main problems in image restoration is to restore the image details smoothed by the blurring process, which are modeled by the point spread function (PSF), but with the compromise of keeping the noise in acceptable levels. This fact oriented the development of iterated image restoration techniques, in which the amount of image restoration can be controlled among the iterations by using regularization or projection techniques. One of the most used projection techniques are the Row-Action Projection Method (RAP) and the Simultaneous Iterative Reconstruction Technique (SIRT) (KUO; MAMMONE, 1992), which were compared in the context of remote sensing image restoration in (PAPA; MASCARENHAS; FONSECA, 2005). Since the image restoration problem can be modeled by a linear system, the RAP method uses a priori knowledge about the image or the imaging system and compute the

projections onto the hyperplanes, which are constrained by a relaxation parameter, iteratively until to meet some criteria (STARK; YANG; YANG, 1998). Both RAP and SIRT techniques belong to the same generically approaches of projection-based algorithms, commonly addressed by Projections Onto Convex Sets (POCS) approaches. The POCS method uses *a priori* knowledge about the image or the imaging system. The key to effectively apply this kind of algorithm is to define the appropriate sets, compute the projection onto these sets, and incorporate the projectors into an image processing algorithm designed to meet some criteria implied by the constraints (STARK; YANG; YANG, 1998).

However, the RAP algorithm for image restoration presented by (KUO; MAMMONE, 1992) have some limitations, and one of the most important is its expensive computational effort. Trying to address this problem, (R.J., 1992) proposed the RAP-2D, which is a modification of the traditional RAP algorithm, which runs much faster and works similar to a convolution process, by using windowing techniques to restore the image. As we know, the windowing techniques suffer from the blocking artifacts production due to the discretization and limited size of the windows, making the RAP-2D particularly extremely dependent of its relaxation parameter. High values can make the restoration process faster, but its more difficult to handle the artifacts growing process, and low values of the relaxation parameter can lead us to a poor image restoration. This make the choice of the relaxation parameter a hard task, and extremely dependent of the blur and noise characteristics.

Methods to find optimal or quasi-optimal solutions (parameters optimization) for problems in image processing based on evolutionary computation have been extensively addressed in the last years. (SEO, 2007) applied Genetic Algorithms (GA) for selection and feature combination in pattern recognition applications. Techniques based on Genetic Programming (GP) (KOZA, 1992) have been used for feature combination in the context of Content-Based Image Retrieval (CBIR) (TORRES et al., 2008). Both GA and GP-based techniques try to find the solution over the natural selection of the possible solutions (individuals) among the iterations of the algorithm (generations). The main difference between GA and GP is the way that the data are modeled: GA-based techniques model the individuals as been strings containing 0 or 1 values, instead of GP approaches, which represent the possible solution with more robust data structures, such that binary trees and linked lists, for instance.

In that way, the main goal of this work is to improve and stabilize the performance of the POCS image restoration algorithm by choosing an optimal or quasi-optimal relaxation parameter through GP-based techniques. In order to validity our method, CBERS-2 CCD band 2 satellite images were restored with the optimal relaxation parameter obtained by GP in Lena phantom images, which were degraded with the same CBERS-2 CCD band 2 blur and noise models. The remainder of the paper is organized as follows. Section 2 presents the image restoration formulation. Sections 3 and 4 contain, respectively, the POCS and GP theory. Section 5 discusses the experimental results and Section 6 provides some conclusions.

2. Image Restoration

The image restoration problem reported here is to obtain an estimate of an image f from its degraded and noisy observation g which is the result of a linear imaging system modeled by

$$g = Hf + n, \quad (1)$$

where H is the convolutional degradation operator, denoted here as the block circulant matrix, and n denotes the additive observation noise (GONZALEZ; WOODS, 2001). Vectors g , f and n correspond to lexicographical ordering of the respective two-dimensional fields by rows, with

dimension M , and columns, with dimension N . Consequently, these vectors and matrix H have, respectively, $MN \times 1$ and $MN \times MN$ dimensions.

The image restoration can be understood as a technique used to correct the distortions produced by the imaging systems. The undesirable effect over the image is detail smoothing and the correction of this problem is based on the sensor characteristics. Therefore, for each satellite, sensor and spectral band an adjusted filter needs to be used. Regularized and projection-based techniques are the most actively pursued approaches to deal with ill-posed problems, because their iterated algorithms allow a better controlling of the restoration processes.

3. POCS Technique

Some problems can be described in terms of convex sets constraints. POCS (Projections onto Convex Sets) methods can be used to find a common vector f which satisfies these constraints, each of which forms a convex set (STARK; YANG; YANG, 1998). If we suppose that we have *a priori* information constraints about the image, this common vector f lies in the intersection of all the convex sets

$$f \in C_0 = \bigcap_{i=1}^m C_i \quad (2)$$

where the i th closed convex set $C_i \in \mathfrak{R}$ denotes the i th constraint or *a priori* knowledge on f and m is the number of those sets.

If the sets $C_i (i = 1, \dots, m)$ are closed and convex, and their intersection, C_0 , is non-empty, the successive projections on the sets will converge to a vector that belongs to this intersection.

This vector can be found by alternatively projecting it onto the convex sets C_i via corresponding projecting operator P_{C_i} as

$$f^{(k+1)} = P_{C_m} P_{C_{m-1}} \dots P_{C_1} f^{(k)}, \quad (3)$$

where P_{C_i} means the projection onto convex set $C_i \in \mathfrak{R}$ in the k th iteration. The initial guess $f^{(0)}$ can be any vector in \mathfrak{R} .

3.1. Convex Restriction Sets

3.1.1. Row-Action Projection - RAP

The image restoration linear model can be described by Equation (4)

$$g = Hf, \quad (4)$$

which is similar to Equation (1), but without the noise term, where each line of g denotes an equation that can be represented by a hyperplane, which is a convex set, and its solution can be found by the POCS method.

If the intersection of these convex sets is non-empty, the main goal is to find this intersection set, which will contain the solution of the restoration problem. The POCS algorithm to obtain the solution of the linear system described by Equation (4) is called Row-Action Projection (RAP) or ART (Algebraic Reconstruction Technique) in tomography, which was initially developed by Kaczmarz in 1937 (R.J., 1992). The POCS method is a generalization of RAP, where the hyperplanes may be substituted by other types of convex sets (KUO; MAMMONE, 1992). The method converges to the hyperplanes intersection, and the RAP equation is given by

$$f^{(k+1)} = f^{(k)} + \lambda \frac{g_p - h_p^t f^k}{\|h_p\|^2} h_p, \quad (5)$$

where λ is the relaxation parameter, g_p is the p th element of vector g , h_p^t is the p th row of matrix H and $f^{(k+1)}$ is the $f^{(k)}$ projection onto the corresponding hyperplane. The iteration index is related to the equation index by $p = k(\text{mod } MN)$, indicating that each row is used multiple times in the restoration process. Thus, by definition, the $P_i x$ projection of a vector x onto C_i set is the point in C_i closest to x . In that way, the $P_i x$ projector on a linear system restriction set, based on Equation (5) is

$$P_i x = P_{C_R} = f^{(k)} + \lambda \frac{g_p - h_p^t f^{(k)}}{\|h_p\|^2} h_p. \quad (6)$$

The success of RAP algorithm implementation depends on the initial condition, iteration number and relaxation parameter λ .

RAP-2D Imaging systems are generally designed so that the degradation matrix H is sparse. In addition, this degradation operator is a block Toeplitz matrix in the shift-invariant case and will represent a 2-D linear convolution given by

$$g(i, j) = \sum_m \sum_n h(i - m, j - n) \hat{f}(m, n). \quad (7)$$

The sparseness of the matrix \mathbf{H} is due to the fact that the size of the PSF is generally much smaller than the size of the image.

The RAP algorithm given by Equation (5) can be implemented by considering only a subregion of the 2-D image \hat{f} that is determined by the size of the 2-D support of the PSF. In this case, every row of the matrix \mathbf{H} in Equation (4) contains only $N \times N$ entries, where N denotes the PSF size. Each pixel $g(i, j)$ of the blurred image g corresponds to a specific equation of the set given by Equation (5). Hence, the 2-D formulation of the RAP algorithm can be written as

$$\hat{f}^{(k+1)}(m, n) = \begin{cases} \hat{f}^{(k)}(m, n) + \lambda \frac{\epsilon(i, j)}{\|h(i, j)\|^2} h(i - m, j - n; i, j), & \text{if } \hat{f}^{(k)}(m, n) \in S_{h(i, j)}, \\ \hat{f}^{(k)}(m, n) & \text{otherwise,} \end{cases} \quad (8)$$

where

$$\epsilon(i, j) = g(i, j) - \sum_{m, n \in S_{h(i, j)}} h(i - m, j - n; i, j) \hat{f}^{(k)}(m, n), \quad (9)$$

$$\|h(i, j)\|^2 = \sum_{m, n \in S_{h(i, j)}} h(m, n; i, j)^2, \quad (10)$$

and $S_{h(i, j)}$ is the support of the PSF centered at pixel $g(i, j)$. In that way, the RAP-2D algorithm can be implemented as a 2-D convolution. That is, each projection operator is local, requiring only the neighborhood $S_{h(i, j)}$ of the image \hat{f} , at each iteration (KUO; MAMMONE, 1992). The RAP-2D projection operator is based on Equation (8), just changing $\hat{f}^{(k+1)}$ of the image by $P_{C_{RAP-2D}}$.

3.1.2. Limited Amplitude Restriction Set

This set describes the upper and lower bounds for the image pixels values to be restored. Equation (11) describes this convex set, where α and β are the lower and upper bounds

respectively and Ω is the support region of the image:

$$C_{LA} = \{s : s \in S \text{ and } \alpha \leq s(k, l) \leq \beta \forall k, l \in \Omega\}, \quad (11)$$

where S is a Hilbert Space. The projection operator onto the set C_{LA} is described as

$$P_{C_{LA}}x(k, l) = \begin{cases} \alpha, & \text{if } x(k, l) < \alpha \\ x(k, l), & \text{if } \alpha \leq x(k, l) \leq \beta \\ \beta, & \text{if } x(k, l) > \beta. \end{cases} \quad (12)$$

3.1.3. Non-negativity Restriction Set

The set given by Equation 13 describes a set in which its elements can not assume negative values, i.e., this set defines a lower bound for the attenuation degree of the image to be restored.

$$C_{NN} = \{s : s \in S \text{ and } s(k, l) \geq 0 \forall k, l \in \Omega\}, \quad (13)$$

The projection operator onto the set C_{NN} is described as

$$P_{C_{NN}}x(k, l) = \begin{cases} 0, & \text{if } x(k, l) < 0 \\ x(k, l), & \text{otherwise.} \end{cases} \quad (14)$$

Since that $C_{NN} \subseteq C_{LA}$ and it is known that the values of an image need to be in the interval $[0, 255]$, in this work we used the following values: $\alpha = 0$ and $\beta = 255$ in Equation 12.

4. Genetic Programming

Genetic algorithms (GAs) (HOLLAND, 1992) and genetic programming (GP) (KOZA, 1992) are a set of artificial intelligence problem-solving techniques based on the principles of biological inheritance and evolution. Each potential solution is called an individual (i.e., a chromosome) in a population. Both GA and GP work by iteratively applying genetic transformations, such as crossover and mutation, to a population of individuals to create more diverse and better performing individuals in subsequent generations. A fitness function is available to assign the fitness value for each individual.

The main difference between GA and GP relies on their internal representation – or data structure – of the individual. In general, GA applications represent each individual as a fixed-length bit string, like (1101110 . . .) or a fixed-length sequence of real numbers (1.2, 2.4, 4, . . .). In GP, on the other hand, more complex data structures are used (e.g., trees, linked lists, or stacks (LANGDON, 1998)). Furthermore, GP data structure length is not fixed, although it may be constrained by implementation to be within a certain size limit. Because of the intrinsic parallel search mechanism and powerful global exploration capability in a high-dimensional space, both GA and GP have been used to solve a wide range of hard optimization problems that oftentimes have no known best solution.

4.1. GP Components

In order to apply GP to solve a given problem, several required key components of a GP system need to be defined. Table 1 lists these essential components along with their descriptions.

The entire combination discovery framework can be seen as an iterative process. Starting with a set of training images with known relevance judgments, GP first operates on a large population of random combination functions (individuals). These combination functions are

Components	Meaning
Terminals	Leaf nodes in a tree structure.
Functions	Non-leaf nodes used to combine the leaf nodes. Commonly numerical operations: +, -, *, /, log.
Fitness Function	The objective function GP aims to optimize.
Reproduction	A genetic operator that copies the individuals with the best fitness values directly into the population for the next generation without going through the crossover operation.
Crossover	A genetic operator that exchanges subtrees from two parents to form two new children. Its aim is to improve the diversity as well as the genetic fitness of the population.
Mutation	A genetic operator that replaces a selected individual's subtree, whose root is a picked mutation point, with a randomly generated subtree.

Table 1: Essential GP Components.

then evaluated based on the relevance information from training images. If the stopping criteria is not met, it will go through the genetic transformation steps to create and evaluate the next generation population iteratively.

GP searches for good combination functions by evolving a population along several generations. Population individuals are modified by applying genetic transformations, such as *reproduction*, *mutation*, and *crossover*. The reproduction operator selects the best individuals and copies them to the next generation. The two main variation operators in GP are mutation and crossover. Mutation can be defined as random manipulation that operates on only one individual. This operator selects a point in the GP tree randomly and replaces the existing subtree at that point with a new randomly generated sub-tree. The crossover operator combines the genetic material of two parents by swapping a sub-tree of one parent with a part of the other.

5. Experimental results

Two rounds of experiments were conducted to demonstrate the validity of the proposed work. In the first series of experiments (Subsection 5.1), we first evaluate the effectiveness of our work in a phantom image, displayed by Figure 1a (a 128×128 , 8 bits/pixel Lena image). This phantom image was degraded with the same CBERS-2 CCD band 2 satellite blur and noise models in order to obtain the optimal λ^* (RAP-2D relaxation parameter). Further, we use λ^* in Equation 8 to restore the real CBERS-2 images (Subsection 5.2). Recall that, for all situations, we execute the POCS image restoration algorithm with 4 iterations. The phantom restored images were quantitatively evaluated through the ISNR (improvement signal to noise ratio). The degraded images were restored using the POCS algorithm below:

$$f^{(k+1)} = P_{C_{RAP-2D}} P_{C_{LA}} f^{(k)}, \quad (15)$$

where $P_{C_{RAP-2D}}$ and $P_{C_{LA}}$ are the projections onto the RAP-2D and limited amplitude constraint sets, respectively.

5.1. Simulation Tests

In this section, we used as phantom the well known Lena image, which was degraded with two kind of blurring models: a gaussian and an average filter. The first ones was modeled as a bidirectional PSF corresponding to the CBERS-2 CCD band 2 PSF specifications. Further, an additive gaussian noise was applied to the blurred images, to complete the degradation process. Figure 1b displays the degraded Lena image using a gaussian PSF with kernel size of 3×3 and

an additive noise with $\sigma = 2$.

As aforementioned, the RAP-2D projector algorithm is extremely sensible with respect to the relaxation parameter λ , in which low values can not fully restore the image (Figure 1c) and high values can degrade the image (Figure 1d). Intermediary values of λ can also produce blocking artifacts in high details regions of the images, as we can see in Figure 1e. Trying to stabilize the RAP-2D algorithm, we proposed here the selection of λ parameter by GP technique, in which its success are dependent of the fitness function F chosen. Here, we used as the fitness function the ISNR value, which means that only λ values that maximizes the ISNR were chosen as the individuals for the next generation of GP algorithm. Equation 16 describes the fitness function used.

$$F = \begin{cases} ISNR & \text{if } ISNR > 0, \\ -\infty & \text{otherwise} \end{cases} \quad (16)$$

The GP algorithm outputs the best individual, which is represented by a binary tree containing some arithmetic operations, in order to obtain λ^* . Figure 1f displays the restored Lena image with the POCS algorithm (Equation 15) and λ^* obtained by GP.



Figure 1: Lena phantom images: (a) Original image. (b) Degraded image. (c) Restored image with $\lambda = 0.1$. (d) Restored image with $\lambda = 3$. (e) Restored image with $\lambda = 1.5$. (f) Restored image with $\lambda^* = 0.315026$.

Table 2 displays the simulation results, in which the Lena image was degraded with a gaussian and an average blurring models and with an additive gaussian noise. Some ISNR results using other randomly selected λ values are also displayed to demonstrate the robustness of the proposed GP-based image restoration algorithm.

Blur model	Gaussian noise variance	PSF size	λ^*	ISNR*	ISNR ($\lambda = -1$)	ISNR ($\lambda = 1$)	ISNR ($\lambda = 3$)
Gaussian	2.0	3×3	0.3150	0.4383	-11.1542	-1.9205	-14.1401
Gaussian	5.0	3×3	0.2987	-0.8144	-12.1377	-5.4255	-15.4934
Average	2.0	5×5	0.0042	-3.2485	-12.8807	-5.7337	-16.8226
Average	5.0	5×5	0.0	-3.5461	-13.3810	-7.7762	-17.2314

Table 2: Simulation results (ISNR* means the ISNR obtained with λ^*)

5.2. Real Tests

In this section we applied the proposed GP-based framework in two real images obtained from CBERS-2 CCD band 2 satellite (Figure 2), which were restored by the POCS algorithm with λ^* obtained over a phantom image, as described in Subsection 5.1. As we can see, good results can be achieved using the GP-based image restoration framework.

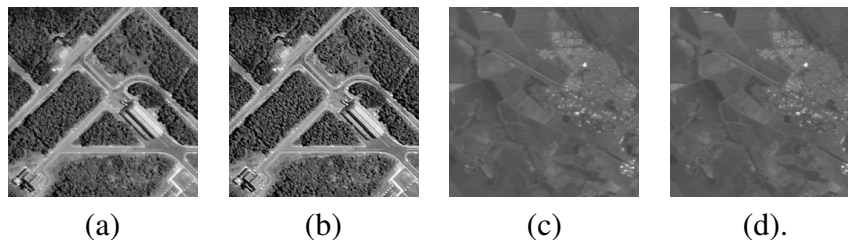


Figure 2: CBERS-2 CCD band 2 satellite images. (a) and (b) are the original and restored images from Alcântara Base, Maranhão-Brazil. (c) and (d) are the original and restored images from São José dos Campos, São Paulo-Brazil.

6. Conclusions

We present here a GP-based approach that stabilizes the POCS image restoration algorithm and the RAP-2D projection operator. Due to the RAP-2D sensitivity to the relaxation parameter, an empirically choice of λ is an inviable task. So, we used GP-based techniques to find the optimal or quasi-optimal λ^* that maximizes the ISNR value for a given image. We intend to direct our studies now for the parameter optimization of other convex restriction sets.

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