SPOT PANCHROMATIC BAND SIMULATION BY LINEAR COMBINATION OF MULTISPECTRAL BANDS

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ABSTRACT

The simulation of one or more bands of a multispectral sensor by combination of other bands represents an attractive possibility. For example, it could decrease the data rate of the communications link between the spacecraft and ground. In this work an analysis of the simulation of the spatially degraded SPOT panchromatic band by linear combination of the multispectral bands has been performed. Two methods have been used for the spectral linear combination: a) unconstrained least squares and b) equality constrained least squares. By using the second method, it is shown that the signal to noise ratio of the simulated band tends to be better than the signal to noise ratio of each multispectral band. The experimental results have included a spatial simulation of a degraded, 20 m resolution panchromatic band for comparison with the spectrally simulated band. These results demonstrate that a numerically and visually reasonable estimate of the degraded panchromatic band can be obtained.

RESUMO

A simulação de uma de mais bandas de um sensor multiespectral por combinação de outras bardas apresenta possibilidades atraentes. Por exemplo, ela poderia diminuir a taxa de dados do canal de comunicação entre o satélite e a Terra. Neste trabalho, é feita uma análise da simulação da banda pancromática do SPOT (degradada espacialmente), por uma combinação linear de bandas multiespectrais. Dois métodos são usados para a combinação linear espectral: a) mínimos quadrados sem restrições e b) mínimos quadrados com restrições de igualdade. Usando o segundo método, mostra-se que a relação sinal-ruído da banda simulada tende a ser melhor que a relação sinal-ruído de cada banda multiespectral. Os resultados experimentais incluem a simulação espacial de uma banda pancromática degradada espacialmente, com uma resolução de 20 m, para comparação com a banda simulada espectralmente. Esses resultados demonstram que uma estimativa razoável do ponto de vista visual e numérico da banda pancromática degradada pode ser obtida.

1. INTRODUCTION

The simulation of satellite imagery is a useful tool as an image processing technique. One could mention at least two applications in the remote sensing area. a) The large sizes of remote sensing images impose a severe burden on the communications link between the satellite and the ground. Therefore, the simulation of an image by ground processing could offer the potential of a decrease on the data rate. b) In sensor feasibility studies it is common practice to develop simulation procedures before the actual sensor is built, so that many technical aspects can be predicted in advance.

The main idea of this paper is to present a simulation technique for a sensor band based on linear combination of other bands. This is possible if there is a substantial spectral overlap between the simulated band and the bands that are linearly combined. As an example of this procedure, we present a study of the simulation of a degraded, 20 m resolution SPOT panchromatic band (P), by linear combination of the two multispectral bands (XS1 and XS2) that spectrally overlap the panchromatic band. The use of the third multiespectral band (XS3) was discarded because of the negligible spectral overlap between P and XS3.

2. THEORETICAL BACKGROUND

Let $f(\lambda)$ be the spectral characteristic of the scene, $g_i(\lambda)$, $i = 1,2$ be the spectral characteristics of the two multispectral bands and $g_p(\lambda)$ the spectral characteristic of the panchromatic band.

The observed value at each pixel of the multispectral bands can be approximately given by:

$$b_i = \int f(\lambda)\, g_i(\lambda) d\lambda \quad i = 1,2 \tag{1}$$

Likewise, for the panchromatic band

$$p = \int f(\lambda)\, g_p(\lambda) d\lambda \tag{2}$$

We want to estimate p by $\hat{p}$, where $\hat{p}$ would be given by a fictitious band that is a linear combination of the two multispectral bands, ie:

$$\hat{p} = \int f(\lambda)\, \hat{g}_p(\lambda) d\lambda \tag{3}$$

where

$$\hat{g}_p(\lambda) = \hat{\alpha}_1 g_1(\lambda) + \hat{\alpha}_2 g_2(\lambda) \tag{4}$$

Therefore

$$\hat{p} = \int f(\lambda)[\hat{\alpha}_1 g_1(\lambda) + \hat{\alpha}_2 g_2(\lambda)]\, d\lambda \tag{5}$$

If the coefficients $\hat{\alpha}_1$ and $\hat{\alpha}_2$ can be determined, the estimate $\hat{p}$ of p at each pixel would be given by

$$\hat{p} = \hat{\alpha}_1 b_1 + \hat{\alpha}_2 b_2 \tag{6}$$

By comparing eqs. (2) and (5) it is clear that we have to approximate the spectral curve $g_p(\lambda)$ by the linear combination $\hat{\alpha}_1 g_1(\lambda) + \hat{\alpha}_2 g_2(\lambda)$. A reasonable criterion to adopt is based on the least squares procedure, that is, we want to minimize

$$\int [g_p(\lambda) - (\alpha_1 g_1(\lambda) + \alpha_2 g_2(\lambda))]^2\, d\lambda \tag{7}$$

by the appropriate choice of α_1 and α_2. In order to do this we discretize the curves $g_p(\lambda)$, $g_1(\lambda)$ and $g_2(\lambda)$ over n points and a linear regression model is obtained

$$\begin{aligned} g_p(\lambda_1) &= \alpha_1 g_1(\lambda_1) + \alpha_2 g_2(\lambda_1) + r_1 \\ g_p(\lambda_2) &= \alpha_1 g_1(\lambda_2) + \alpha_2 g_2(\lambda_2) + r_2 \\ &\vdots \\ g_p(\lambda_n) &= \alpha_1 g_1(\lambda_n) + \alpha_2 g_2(\lambda_n) + r_n \end{aligned} \tag{8}$$

The scalars r_i, $i = 1, 2 \ldots n$ take into account the error in the approximation.

By defining

$$\underline{g}_p = \begin{bmatrix} g_p(\lambda_1) \\ g_p(\lambda_2) \\ \vdots \\ g_p(\lambda_n) \end{bmatrix}_{n \times 1} \qquad \underline{G} = \begin{bmatrix} g_1(\lambda_1) & g_2(\lambda_1) \\ g_1(\lambda_2) & g_2(\lambda_2) \\ \vdots & \vdots \\ g_1(\lambda_n) & g_2(\lambda_n) \end{bmatrix}_{n \times 2} \tag{9}$$

$$\underline{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}_{2 \times 1} \qquad \underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}_{n \times 1}$$

We obtain the linear model

$$\underline{g}_p = \underline{G}\,\underline{\alpha} + \underline{r} \tag{10}$$

It is well known that under the hypothesis that the rank of the matrix $\underline{G}$ is given by the number of columns (2 in this case), the least squares solution of this overdetermined system is given by

$$\underline{\hat{\alpha}} = (\underline{G}^T \underline{G})^{-1} \underline{G}^T \underline{g}_p = \underline{G}^+ \underline{g}_p \tag{11}$$

where $\underline{G}^+$ is the Moore-Penrose pseudoinverse of $\underline{G}$ (Lewis and Odell, 1971).

For reasons that will become clear in the next section, one may want to impose an additional constraint on the values of α_1 and α_2, namely that $\alpha_1 + \alpha_2 = 1$. In matrix form, this can be expressed as

$$\underline{A}\ \underline{\alpha} = t \tag{12}$$

$$\text{where } A = (11) \tag{13}$$

$$t = 1 \tag{14}$$

This linear equality constrained least squares problem has the solution given by (Lewis and Odell, 1971):

$$\tilde{\underline{\alpha}} = \hat{\underline{\alpha}} + (\underline{G}^T\underline{G})^{-1}\ \underline{A}^T[\underline{A}\ (\underline{G}^T\underline{G})^{-1}\ \underline{A}^T]^{-1}(t-\underline{A}\hat{\underline{\alpha}}) \tag{15}$$

where $\hat{\underline{\alpha}}$ is given by equation (11).

3. SIGNAL TO NOISE RATIO CALCULATION

Let us assume that the linear constraint given by eq. (12) is imposed. In practice, it was verified the components $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are positive so no inequality constraints have to be imposed.

In order to calculate the signal to noise ratio of the simulated band and compare with the signal to noise ratio of the bands b and b , let us assume that each of these bands is degraded by additive noise, ie

$$b_1 = s_1 + n_2 \tag{16}$$

$$b_2 = s_2 + n_2 \tag{17}$$

Assume for simplicity (without any loss of generality) the following conditions:

$$E(s_1) = E(s_2) = E(n_1) = E(n_2) = 0 \tag{18}$$

$$E(n_1^2) = E(n_2^2) = \sigma_n^2 \tag{19}$$

$$E(s_1^2) = E(s_2^2) = \sigma_s^2 \tag{20}$$

Furthermore, let us also assume uncorrelated noise processes on the two bands, is

$$E(n_1\ n_2) = 0. \tag{21}$$

For each multispectral band

$$SNR_b = \frac{\sigma_s^2}{\sigma_n^2} \tag{22}$$

For the simulated band given by

$$\tilde{p} = \underbrace{\tilde{\alpha}_1 s_1 + \tilde{\alpha}_2 s_2}_{s_p} + \underbrace{\tilde{\alpha}_1 n_1 + \tilde{\alpha}_2 n_2}_{n_p} \tag{23}$$

we have

$$SNR_{\tilde{p}} = \frac{\sigma_{sp}^2}{\sigma_{np}^2} \tag{24}$$

We need an additional assumption which is frequently verified in practice, namely high correlation between multispectral bands, ie

$$\rho_{s_1 s_2} \cong 1 \tag{25}$$

Then, it can easily be shown that

$$SNR_p = \frac{\sigma_s^2}{\sigma_n^2(1-2\tilde{\alpha}_1\tilde{\alpha}_2)} \tag{26}$$

But $1 - 2\tilde{\alpha}_1\tilde{\alpha}_2 < 1$ since $\tilde{\alpha}_1$ and $\tilde{\alpha}_2 > 0$. Therefore

$$SNR_{\tilde{p}} > SNR_b \tag{27}$$

This result shows that an improved signal to noise ratio is obtained in the simulated band. Let us remark that the key assumptions leading to this derivation are: 1) linear constrained estimators ($\tilde{\alpha}_1 + \tilde{\alpha}_2 = 1$) and 2) high correlation between the combined bands b_1 and b_2. It should be observed that in the experimental results we obtained a correlation coefficient of .89 between b_1 and b_2.

4. SIMULATION USING SPOT IMAGES

The use of only XS1 XS2 SPOT multispectral bands to simulate a spatially degraded SPOT panchromatic band is justified by the fact that XS3 has little overlap with the P-band (see Figure 1).

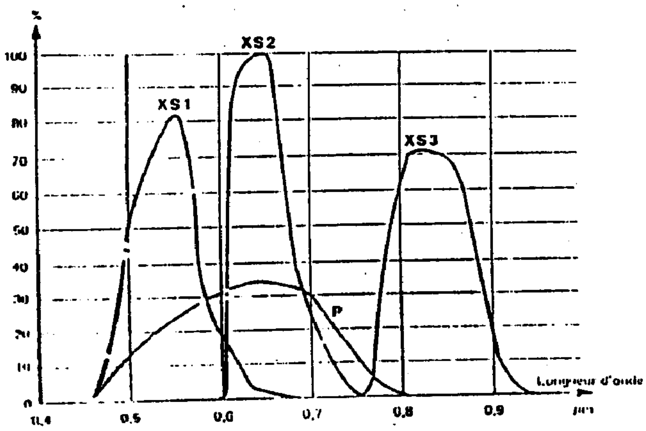

Figure 1 - Typical Spectral Sensitivity of HRV Instruments (source: CNES; SPOT-Image, 1986).

The spectral curves of XS1, XS2 and P were discretized with 10 points (ie, n = 10), leading to the following numerical results for the unconstrained solution

$$\hat{\alpha} = \begin{bmatrix} 0.3426 \\ 0.397 \end{bmatrix}$$

and the constrained solution

$$\alpha = \begin{bmatrix} 0.4937 \\ 0.5046 \end{bmatrix}$$

Figure 2 shows the discretized XS1 and XS2 curves. Figure 3 displays the discretized P curve, the unconstrained estimated $\hat{P}$ curve and the linear constrained estimated $\tilde{P}$ curve.

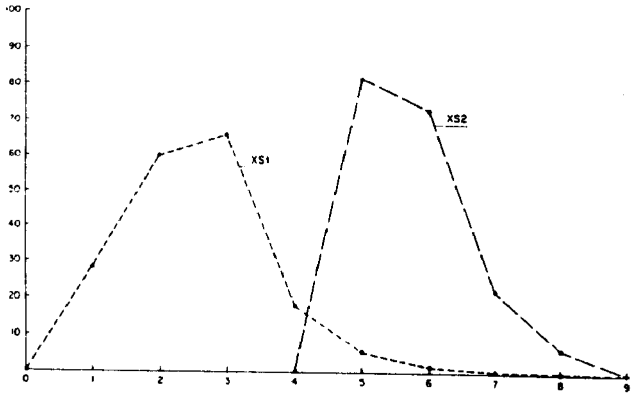

Figure 2 - Discretized XS1 and XS2 curves.

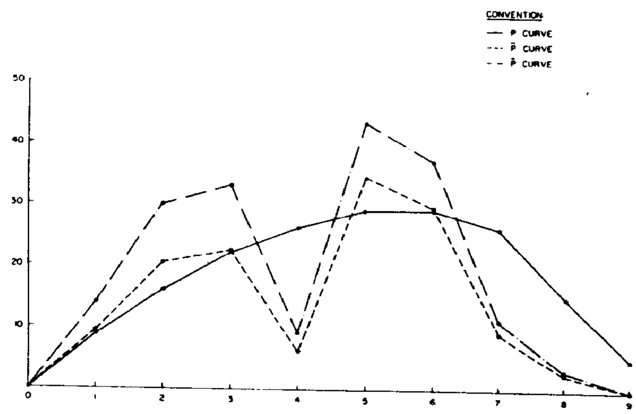

Figure 3 - Discretized P, $\hat{P}$ and $\tilde{P}$ curves.

5. EXPERIMENTAL RESULTS

5.1 - Spatial Simulation of 20 m - Resolution SPOT Panchromatic Band

Since bands XS1 and XS2 have a sampling rate of 20 m, we degraded the 10 m P band, according to the following two steps

a) Step 1 - Spatial Filtering of the 10 m Resolution Panchromatic Band

The P-band was spatially degraded by applying twice the low pass linear filter with finite impulse response given by

$$\frac{1}{3000}\begin{pmatrix} 169 & 337 & 169 \\ 412 & 826 & 412 \\ 169 & 337 & 169 \end{pmatrix}$$

This impulse response was obtained following the procedure suggested by (Banon, 1990) and considering the attenuation factor γ of the modulation transfer function at half the sampling rate (that is, at Nyquist frequency) given in Table 1

Band	P	XS
row	0.27	0.21
column	0.16	0.21

Table 1 - Atenuation Factor γ

The values for P-band are those of (CNES; SPOT Image, 1986); the values for XS2-band are not those of the previous reference but have been reevaluated so that the visual impression of the simulated panchromatic band be similar to the multispectral bands from the point of view of spatial resolution.

b) Step 2 - Resampling of the Filtered 10 m Resolution Panchromatic Band

The filtered 10 m resolution panchromatic band of step 1 was resampled at 0.5 spatial rate, ie, keeping only the first pixel of each pair of consecutive pixels along the same line and keeping only the first line of each pair of consecutive lines. Figure 6 shows the result of the spatial simulation of the 20 m resolution panchromatic band.

It should be noted that the filtering and resampling procedures could also be done using multirate digital signal processing techniques, according to (Fonseca; Mascarenhas, 1988).

5.2 - Spectral Simulation of 20 m - Resolution Panchromatic Band

5.2.1 - Unconstrained Simulation

The spectrally simulated image has lower mean and lower variance than the spatially simulated band. The reason for this lies in the fact that the spectral curves do not incorporate offset and gain factors that are used in ground processing.

5.2.2 - Constrained Simulation

It was observed that the histogram of the spectrally simulated image under a linear constraint is closer to the histogram of the spatially simulated image than in the previous case; however, mean and variance of spectrally simulated image are still lower (although closer) than the spatially simulated image.

5.2.3 - Gain and Offset Adjustment

The adopted procedure is to introduce the gain and offset adjustment so that the mean value and the standard deviation of the estimated band (either through the unconstrained or the constrained estimation) are equal to the spatially simulated 20 m resolution band, ie, in the unconstrained case:

$$\hat{p}' = \alpha[\hat{\alpha}_1 b_1 + \hat{\alpha}_2 b_2] + \beta$$

such that

$$E\,[\hat{p}'] = E\,[p^0]$$

$$\sigma[\hat{p}'] = [p^0]$$

where p^0 is the spatially simulated panchromatic band. Analogous expressions are valid for the constrained estimators. In order to avoid increasing round off errors $\alpha\hat{\alpha}_1$ and $\alpha\hat{\alpha}_2$ were combined by applying the transformation directly on b_1 and b_2. It was verified that the histogram of the gain and offset compensated estimator without constraint approximates better the histogram of the spatially degraded panchromatic band than the constrained estimator. Figures 4 and 5 allows the comparison between the histograms for the former case.

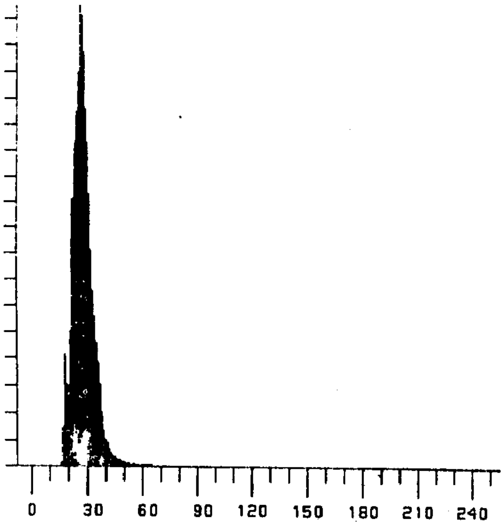

Figure 4 - Histogram of spatially simulated P-band.

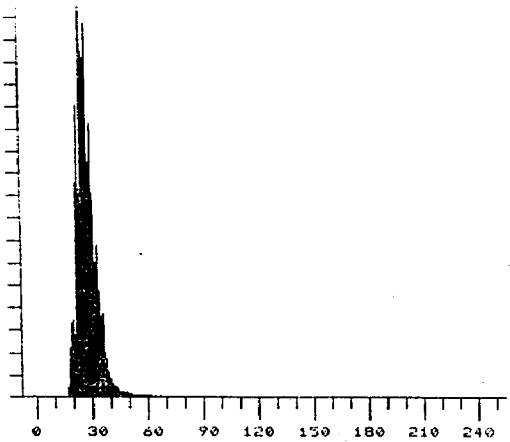

Figure 5 - Histogram of gain and offset -compensated spectrally simulated panchromatic band without constraint.

5.2.4 - Mean Square Error Between Images

The m.s.e. between the spatially simulated band (P_2) and the spectrally simulated bands without constraint (P_{2_S}), with constraint (P_{2_C}), without constraint with gain and offset compensation ($P_{2_{SG}}$) and with constraint with gain and offset compensation ($P_{2_{CG}}$) were computed, according to the expression

$$d = \left\{\frac{1}{(256)^2} \cdot \sum_{i=1}^{256} \sum_{j=1}^{256} [X(i,j) - Y(i,j)]^2\right\}^{1/2}$$

These results are displayed on Table 2.

d	P
P_{2S}	10.75
P_{2C}	4.87
P_{2SG}	2.65
P_{2CG}	2.68

Table 2 - Mean Square Error Between Spatially and Spectrally Simulated Bands

One can observe that: a) P_{2C} approximates P_2 better than P_{2S}; b) the results for P_{2SG} and P_{2CG} are approximately the same.

5.2.4 - Visual Results

Figure 6 displays 256 x 256 images of P_2 P_{2S} and P_{2C}. Figure 7 displays P_2, P_{2SG}, P_{2CG} and the absolute value of the difference image (P_2 - P_{2SG}), through a linear constrast stretch that increased the maximum value of the absolute value of the difference image from approximately 16 to 85 for visual purposes. Figure 8 displays the SPOT multispectral bands XS1(b_1) and XS2(b_2).

It is observed that P_{2S} has lower constrast than P_2 due to unknown gain and offset factors. P_{2C} approximates P_2 better than P_{2S}, which is in accordance with Table 2. Furthermore, P_{2SG} and P_{2CG} are very close to P_2 in visual terms.

6 - CONCLUSIONS

Through least squares criteria the possibility of simulating one band by linear combination of bands that spectrally overlap the desired band has been demonstrated. Numerically and visually acceptable results were obtained.

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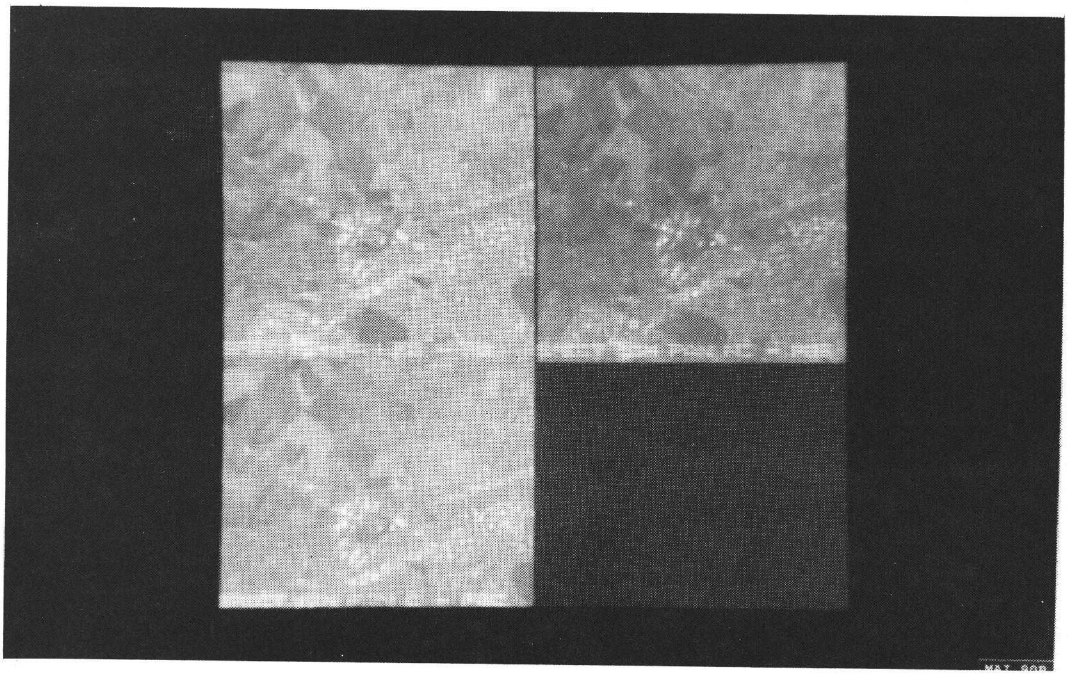

Figure 6 - Images of P_2 , P_{2S} and P_{2C}.

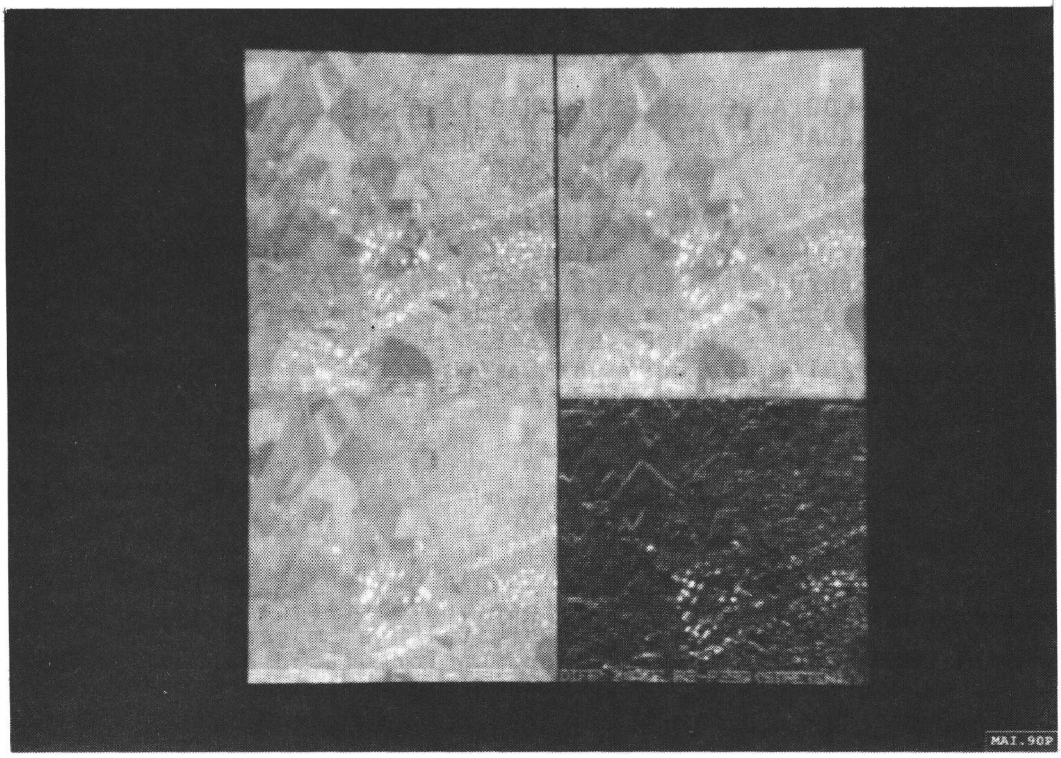

Figure 7 - Images of P_2, P_{2SG} , P_{2CG} and $|P_2 - P_{2SG}|$.

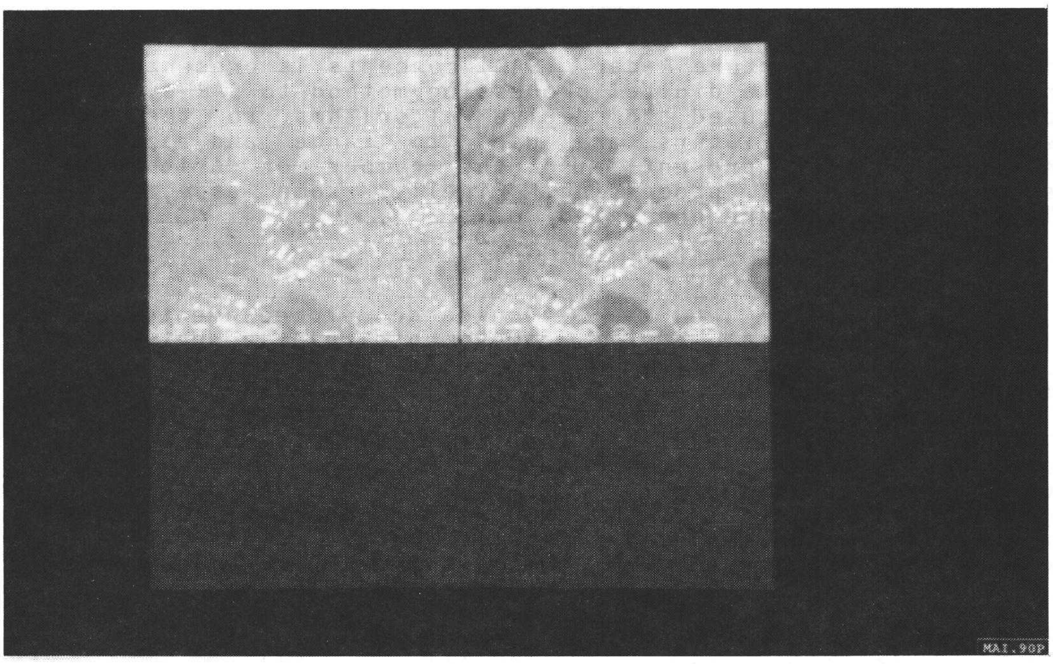

Figure 8 - Images of XS1 and XS2.