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Estimation of spectrum from speckled SAR images

OSCAR HUMBERTO BUSTOS[1]
ANA GEORGINA FLESIA[1]

[1]FAMAF--Facultad de Matemática, Astronomía y Física
Estafeta Ciudad Universitaria, Córdoba, Argentina
{bustos,flesia@mate.uncor.edu}

Abstract The multiplicative model can be used to describe SAR image formation. In this context, the effect and nature of coherent speckling on the spectrum of SAR images is investigated. A method for estimate the spectrum of the backscatter image, based on estimates of the spectra of the speckled image and noise is developed.

1 Introduction

Images and signals produced by coherent systems are subject to the phenomena of speckle. This kind of noise appears due to interference phenomena between the incident and reflected signals. The result makes visual and automatic interpretation a difficult task, thought it may carry some important information.

Usually, images suffering from speckle noise should not be treated with the usual additive-noise derived tools (Wiener filter, for instance), since speckle corrupts the signal in the multiplicative manner and in the amplitude and intensity formats it is non-gaussian [Goodman (1976), Tur et al.(1982)].

Other schemes have been proposed to deal with it, such multilook processing (incoherent average), or various types of linear and adaptative filters. These efforts have generally been directed toward improvement of the signal in the image domain.

However, application exist in which the spectrum of the output is of primary interest, [Beal (1980)], but even in linear filtering aimed at image improvement, it would be useful to have a good estimate of the underlying image spectrum rather than work from a priori assumption such as is often done.

This paper extends one-dimensional results on the problem of estimating the spectrum for speckled data [Goldfinger (1982)]. We first discuss the effects of speckle in SAR images and the mathematical framework used to explain the statistical behaviour of this kind of data. Then we prove that, as long as the output Z is stationary, the power spectral density of Z will be a convolution of the power spectral densities of the backscatter X and the speckle Y in the intensity format. In view of that, we consider three special cases. These cases are uniform target, white uncorrelated speckle and nearest neighbour correlation.

The last section presents an estimate of the underlying spectrum based on classical estimates of the return and noise spectrum. We shall study the performance of that estimate based on the performance of the other estimates involved in making it.

2 Notational conventions and general definitions

The set of real numbers is denoted by **R,** the set of natural numbers by **N,** and the set of integers numbers by **Z**. The generics points of the two-dimensional set of integers $\mathbf{Z^2}$ are denoted by (s_1,s_2), where s_1 always represents the horizontal coordinate, and s_2 the vertical coordinate.

Random processes will be denoted by $X = \{X(s_1,s_2)/(s_1,s_2) \in Z^2\}$. A common underlying probability space will be assumed throughout this work *(W, A , P)*, where *W* denotes the sample set, *A* its *s*-algebra, and *P* a probability. Therefore, real or complex-valued random processes are collections of measurable functions of the form $X(s_1,s_2)$*: W®**R*** or $X(s_1,s_2)$*: W®**C***, indexed in $\boldsymbol{Z^2}$**.**

Let's *k* ***Î*** ***Z*** and *N* ***Î*** **N**, we denote by $w_{k,N}$ the N^{th} unit's root

$$w_{k,N} = \exp\left(\frac{2pk}{N} i\right)$$

They have the following properties

[a)] $w_{0,N} + \ldots + w_{N-1,N} = 0$, since $w_{k,N}$ with $0 \le k \le N-1$ are the roots of the equation $z^N - 1 = 0$.

[b)] $w_{k+rN,N} = w_{k,N}$ for all *r* ***Î*** ***Z*** and *0 £ k £ N-1.*

[c)] $w_{k,N}^{*} = w_{-k,N} = w_{N-k,N}$ for all *0 £ k £ N-1.*

[d)] $w_{p0,N} + \ldots + w_{p(N-1),N} = N$ if *p=0* , and *0* if *1£ p £ N-1*, with *p* ***Î*** ***N***.

3 The multiplicative model and the speckle noise

The multiplicative model has been widely used in the modelling, processing, and analysis of synthetic aperture radar images. This model states that, under certain conditions [Goodman (1976) , Tur et al. (1982)], the return results from the product between the speckle noise and the terrain backscatter.

Based upon this model, we assume that the observed value in each pixel within this kind of images is the outcome of the product of two independent two-dimensional random processes: one *X* modelling the terrain backscatter, and other *Y* modelling the speckle noise. The former is many times considered real and positive, while the latter could be complex (if the considered image is in the complex format) or positive and real (intensity and amplitude formats).

Therefore, the observed value is the outcome of the random process defined by the product

$$Z(s_1,s_2) = X(s_1,s_2)\, Y(s_1,s_2) \qquad \text{forall} \qquad (s_2,s_2) \in \mathbf{Z}^2, \qquad (1)$$

where (s_1,s_2) denotes the spatial position of the pixel.

We will say that the process Z_I is the intensity return process if $Z_I = |Z|^2$, and Z_A is the amplitude return process if $Z_A = |Z|$.

The complex format has been used as a flexible tool for the statistical modelling of SAR data. However, in several cases, complex data are not available or it exists computational limitations imposed by the imaging system that not allow us to work with them. In order to that, intensity format and amplitude formats are frequently considered in the literature.

In many cases, it is easier to derive the statistical properties of the intensity data rather than amplitude data. For instance, the intensity speckle noise modelled as the sum of independent and exponentially distributed random variables has well know distribution, the Gamma distribution, but this is not the case for amplitude speckle noise, since the convolution of Raylegh distributions has not closed form. [Yanasse (1995)]

In this work, we want to estimate the spectrum of the backscatter based on the spectrum of the return, in the intensity format.

Following the description that Frery et al. (1997) realise about the appropriated distributions for this model, complex speckle is assumed to have a bivariate normal distribution, with independent identically distributed components having zero mean and variance *1/2*. These marginal distributions are denoted here as $N(0,1/2)$, therefore, $Y_C(s_1,s_2) = (Re(Y(s_1,s_2)), Im(Y(s_1,s_2)))\sim N^2(\mathbf{0},\mathbf{1/2})$ denotes the distribution of a pair.

Multilook intensity speckle results from taking the average over n independent samples of $Y_{I,}(s_1,s_2)=|Y_{C,}(s_1,s_2)|^2$ leading, thus, to a Gamma distribution denoted here as $Y_{I,}(s_1,s_2) \sim G(n,n)$ and characterised by the density

$$f_{Y_I} = \frac{n^n}{\Gamma(n)} y^{n-1} \exp(-ny) \;\; y > 0,\; n > 0. \qquad (2)$$

Several distributions could be used for the backscatter, aiming at the modelling of different types of classes and their characteristic degrees of homogeneity. For instance, for some sensor parameters (wavelength, angle of incidence, polarisation, etc), pasture is more homogeneous than forest, which in turn are more homogeneous than urban areas.

The basic hypothesis that governs the modelling of homogeneous regions is that the backscatter is constant, thought its value is unknown. When the region is non-homogeneous, the backscatter can be modelled for a more convenient distribution.

The distribution of the intensity return arises from the product $Z_I=X_I.Y_I$. For instance, in the homogeneous case, we consider X_I a constant b and the multilook intensity speckle $Y_I(s_1,s_2)\sim G(n,n)$, then the return Z_I can be modelled by a Gamma distribution, denoted by $Z_I(s_1,s_2)\sim G(n,n/b^2)$.

4 Periodic processes and discrete Fourier series

We wish to derive some properties of the spectrum of a random process present in an image that has been speckled by the multiplicative manner described above. Since the data are usually only available in discrete regions, we can restrict our attention to periodic random processes, which we define below.

A complex two-dimensional random process X is said to be periodic with period of N_1 x N_2 when $X(s_1,s_2) = X(s_1+N_1,s_2) = X(s_1,s_2+N_2)$ for all $(s_1,s_2)\in\mathbf{Z}^2$. Since $X(s_1,s_2) r_1^{-s_1} r_2^{-s_2}$ is not absolutely summable for any r_1, r_2, neither the Fourier transform or the Z-transform uniformly converges for periodic processes.

The discrete Fourier series (**DFS**) is a frequency domain representation of a periodic process. The sequence of **DFS** coefficients of X, whose are denoted by $\hat{X}(s_1,s_2)$, is determinated for the equation (3)

$$\hat{X}(s_1,s_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X(k_1,k_2) w_{s_1 k_1, N_1} w_{s_2 k_2, N_2} \quad \forall\, 0 \le s_1 \le N_1 - 1 \quad 0 \le s_2 \le N_2 - 1 \qquad (3)$$

Then, the power spectral density, (**psd**), of the signal X is the sequence defined by

$$S_X(s_1,s_2)=E\left(\hat{X}(s_1,s_2)^*\hat{X}(s_1,s_2)\right) \qquad (4)$$

where $E(.)$ represents the mathematical expectance.

If we suppose that X is stationary in the wide sense we have

$$E\left(X(s_1,s_2)^*X(s_1,s_2)\right)=R_X(s_1-t_2,s_2-t_2) \quad \forall(s_1,s_2)(t_1,t_2)\in Z^2 \qquad (5)$$

where R_X is the auto-correlation function of X.

Expanding the product $\hat{X}^*_{(s_1,s_2)}\ \hat{X}_{(s_1,s_2)}$, and using the stationarity, we obtain

$$E\left(\hat{X}(s_1,s_2)^*\hat{X}(t_1,t_2)\right)=\left(\frac{1}{N_1N_2}\right)^2\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1}R_X(k_1-l_1,k_2-l_2)\,w_{k_1s_1-l_1t_1,N_1}w_{k_2s_2-l_2t_2,N_2} \qquad (6)$$

For a real stationary periodic random process, making appropriated changes of variable and noting that the periodicity implies that all arithmetic is modulo N_1 or N_2 , we can transform the equation, yielding

$$E\left(\hat{X}(s_1,s_2)^*\hat{X}(t_1,t_2)\right)=0 \quad \text{if} \quad (s_1,s_2)\neq(t_1,t_2) \qquad (7)$$

and

$$E\left(\hat{X}(s_1,s_2)^*\hat{X}(t_1,t_2)\right)=\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1}R_X(k_1,k_2)\,w_{k_1s_1,N_1}w_{k_2s_2,N_2} \quad \text{if} \quad (s_1,s_2)=(t_1,t_2) \qquad (8)$$

Then, putting (8) into (4), we obtain that

$$S_X(s_1,s_2)=\frac{1}{N_1N_2}\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1}R_X w_{k_1s_1,N_1}w_{k_2s_2,N_2} \qquad \forall(s_1,s_2)\in Z^2 \qquad (9)$$

5 Effect of speckling

Consider the intensity return process Z_I given by

$$Z_I(s_1,s_2)=X_I(s_1,s_2)Y_I(s_1,s_2)$$

where X_I and Y_I are real periodic independent stationary random processes, with the same period N_1 x N_2. Let's $\hat{X}_I$, $\hat{Y}_I$ and $\hat{Z}_I$ the respective **DFS** of the processes X_I, Y_I, and Z_I.

Given the relation under the random processes X_I, Y_I, and Z_I, it is not difficult to prove that

$$S_{Z_I}(s_1,s_2)=\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\sum_{l_1=0}^{N_1-1}\sum_{l_2-1}^{N_2-1}E\left(\hat{X}_I(k_1,k_2)^*\hat{X}_I(l_1,l_2)\right)E\left(\hat{Y}_I(s_1-k_1,s_2-k_2)^*\hat{Y}_I(s_1-l_1,s_2-l_2)\right) \qquad (10)$$

since the **DFS** of a product process is the convolution, (see Lim (1990)), and X e Y are independent processes.

Then, we can insert (7) and (9) into (10) to get

$$S_{Z_I}(s_1,s_2)=\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\left(\hat{X}_I(k_1,k_2)*\hat{X}_I(k_1,k_2)\right)E\left(\hat{Y}_I(s_1-k_1,s_2-k_2)*\hat{Y}_I(s_1-k_1,s_2-k_2)\right)$$

Thus

$$S_{Z_I}(s_1,s_2)=\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}S_{X_I}(k_1,k_2)S_{Y_I}(s_1-k_1,s_2-k_2) \qquad (11)$$

and, as long as X_I and Y_I are stationary, the power spectral density of Z_I will be the convolution of the power spectral density of X_I and Y_I .

Let us assume that a_Y:$\mathbf{Z^2}\rightarrow\mathbf{C}$ is the normalized auto-correlation function of the noise process Y_I. Then

$$a_Y(s_1,s_2)=\frac{R_{Y_I}(s_1,s_2)-E\left(Y_I(0,0)\right)^2}{\left[R_{Y_I}(0,0)-E\left(Y_I(0,0)\right)\right]^2}=\frac{R_{Y_I}(s_1,s_2)-E(Y_I)^2}{Var\left(Y_I(0,0)\right)} \qquad (12)$$

Notice that, if *Y~G(n,n)*, it holds that $Y_I(s_1,s_2)=1$ and $Var\left(Y_I(s_1,s_2)\right)=1/n$, therefore $R_{Y_I}(s_1,s_2)=1/n\left(a_Y(s_1,s_2)+1\right)$ and

$$S_Y(s_1,s_2)=\frac{1}{n}\hat{a}_Y(s_1,s_2)+d(s_1,s_2) \qquad (13)$$

where δ is the periodic Dirac delta with period of N_1 x N_2 .Using the relation (13) and the definition of **DFS**, (11) becomes

$$S_{Z_I}(s_1,s_2)=S_{X_I}(s_1,s_2)+\frac{1}{n}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}S_{X_I}(k_1,k_2)\,\hat{a}_Y(s_1-k_1,s_{2\_}k_2) \qquad (14)$$

We will consider three special cases.

Case 1: Uniform target

If the target is uniform, the power spectrum of the intensity backscatter can be modelled by

$$S_{X_I}(s_1,s_2)=b^2 d(s_1,s_2)$$

where b^2 is a constant.

Then, the spectrum of the speckled image is given by

$$S_{Z_I}(s_{1,}s_2)=b^2 d(s_{1,}s_2)+\frac{1}{n}\hat{Y}_I(s_{1,}s_2) \qquad (15)$$

Since that equation can be solve to give $\hat{a}_Y$ in terms of S_{Z_I} , it is posible to determine the speckle process auto-correlation function when the target is known to be uniform. The significance of this lies in the fact that the speckle correlation is often produced by the imaging system being used, and thus, this property of the system can be studied by deliberaty viewing a uniform target.

A similar one-dimension technique has been used in the analysis of SEASAT-A image spectra (Beal et al. (1981)).

Case 2: White uncorrelated speckle

In this case, we have $a_Y(s_1,s_2)=\delta(s_1,s_2)$, and $\hat{Y}_I(s_1,s_2)=1/N_1N_2$. Therefore,

$$S_{Z_I}(s_1,s_2)=S_{X_I}(s_1,s_2)+\frac{1}{nN_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}S_{X_I}(k_1,k_2) \qquad (16)$$

We let $\bar{s}_{X_I}(s_1,s_2)=1/N_1N_2\sum_{k_1}\sum_{k_2}S_{X_I}(k_1,k_2)$ the average spectral power, then

$$S_{Z_I}(s_1,s_2)=S_{X_I}(s_1,s_2)+\frac{1}{n}\bar{s}_{X_I} \qquad (17)$$

Therefore, the power spectral density (**psd**) of the speckled signal is proportional to that of the unspeckled signal, but has added to it a bias proportional to the average **psd**. Hence, a **psd** that is sharply peaked will stand out more strongly against the bias than will one that is broader.

Case 3: Nearest Neighbour correlation

We assume that nearest neighbour points are correlated to some extent. That is, we let

$$a_Y(s_1,s_2)=d\,(s_1,s_2)+e\,(d\,(s_1-1,s_2)+d\,(s_1+1,s_2)+d\,(s_1,s_2-1)+d\,(s_1,s_2+1)) \qquad (18)$$

In order to this, we can prove that the **DFS** of the normalized autocorrelation function is given by

$$\hat{a}_Y(s_1,s_2)=\frac{1}{N_1N_2}\left(1+2e\,\cos\left(\frac{2p\,s_1}{N_1}\right)+2e\,\cos\left(\frac{2p\,s_2}{N_2}\right)\right) \qquad (19)$$

Putting equation (19) into equation (14) we obtain

$$S_{Z_I}(s_1,s_2)=S_{X_I}(s_1,s_2)+\frac{1}{n}\bar{s}_{X_I}+\frac{2e}{nN_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_{21}-1}S_{X_I}(k_1,k_2)\left(\cos\left(\frac{2p\,(s_1-k_1)}{N_1}\right)+\cos\left(\frac{2p\,(s_2-k_2)}{N_2}\right)\right)$$

a relationship much more complicated than of (17).

It is important to note that, while uncorrelated speckle merely added a bias to the spectrum, correlated speckle add power that is not uniform in wavenumber, thus changing the shape of the spectrum.

The question of how the underlying spectrum is best to be estimated is taken up in the next section.

6 Estimation of spectrum

To estimate the spectrum S_{X_I} from the speckled spectrum S_{Z_I}, it is necessary to invert (14). We know that, for *n*-look intensity speckle process Γ distributed, the **psd** of the intensity return is given by

$$S_{Z_I}(s_1,s_2) = S_{X_I}(s_1,s_2) + \frac{1}{n}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} S_{X_I}(k_1,k_2)\hat{a}_Y(s_1-k_1,s_2-k_2) \qquad (21)$$

Then, defining

$$\mathbf{S}_{X_I}(k_1,k_2) = \frac{1}{N_1N_2}\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1} S_{X_I}(l_1,l_2) w_{l_1k_1,N_1} w_{l_2k_{21},N_2} \qquad (22)$$

and using the definition of Fourier transform, (21) becomes

$$S_{Z_I}(s_1,s_2) = S_{X_I}(s_1,s_2) + \frac{1}{n}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} \mathbf{S}_{X_I}(k_1,k_2) a_Y(k_1,k_2) w_{s_1k_1,N_1} w_{s_{21}k_2,N_2} \qquad (23)$$

Let is now

$$\mathbf{S}_{Z_I}(k_1,k_2) = \frac{1}{N_1N_2}\sum_{l_1=0}^{N_1-1}\sum_{l_2=0}^{N_2-1} S_{Z_I}(l_1,l_2) w_{l_1k_1,N_1} w_{l_2k_{21},N_2} \qquad (24)$$

Then, multipling both side of the equation (23) by $\exp(2pi\, l_1k_1/N_1)\exp(2pi\, l_2k_2/N_2)$ and summing the result over all *(k₁,k₂)* in the period, we obtain

$$\mathbf{S}_{Z_I}(k_1,k_2) = n^2\left[1 + \frac{1}{n} a_Y(k_1,k_2)\mathbf{S}_{X_I}(k_1,k_2)\right] \qquad (25)$$

Putting the last equation into (22) and solving for S_{X_I} we obtain the desired spectral estimate:

$$S_{X_I}(s_1,s_2) = \frac{1}{n^2}\left[S_{Z_I}(s_1,s_2) - \sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\frac{a_Y(k_1,k_2)}{n + a_Y(k_1,k_2)}\mathbf{S}_{Z_I}(k_1,k_2) w_{s_1k_1,N_1} w_{s_2k_{21},N_2}\right] \qquad (26)$$

Equation (26) is our general result. We consider two special cases.

Case 1: White uncorrelated speckle

In this case, a straightfoward procedure, using the fact that $a_Y(s_1,s_2)=d\,(s_1,s_2)$, allow us that

$$S_{X_I}(s_1,s_2) = \frac{1}{n^2}\left[S_{Z_I}(s_1,s_2) - \frac{1}{1+n}\bar{s}_{Z_I}\right] \qquad (27)$$

We must note that we could also have derived directly this result from (17). Thus estimation of the spectrum on the case of uncorrelated speckle is accomplished by substraction of an appropriate bias.

Case 2: Nearest neighbour correlation

Also in this case, a straightfoward procedure allow us that

$$S_{X_I}(s_1,s_2)=\frac{1}{n^2}\left[S_{Z_I}(s_1,s_2)-\frac{\bar{s}_{Z_I}}{1+n}-\frac{2e}{e+n}\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_1=0}^{N_1-1}S_{Z_I}(s_1,s_2)\right]\cos\left(\frac{2p(s_1-k_1)}{N_1}\right)\cos\left(\frac{2p(s_2-k_2)}{N_2}\right)$$

Therefore, a priori knowledge of the speckle auto-correlation function, such as is available through the study of uniform targets, allows the underlying image spectrum to be estimated. The correction is, however, more complicated than merely the subtraction of a bias.

7 Conclusions

The technique presented in this paper provides an exact representation of the power spectral density for the backscatter process. However, this representation depends strongly on the **psd** of the speckled process, which is unknown. Thus, the estimation of the underlying spectrum is accomplished by the election of an appropriate estimate for the spectrum of the speckled signal, in the most of cases.

We can choose conventional estimates, based on the Fourier transform, like the periodogram, or the class of smoothed periodograms, but for small N_1 and N_2 the resolution of them can be rather poor. It exists other spectral estimation methods based on maximum likerlihood or maximum entropy which give higher resolution than the conventional estimates. However, since the various methods were developed on the basis of different assumptions, and since only limited comparisons of the method’s performance are available, choosing the best method for a given application problem is a difficult task.

8 References

Beal,R.C. (1980) *Spaceborne imaging radar; monitoring ocean waves.* Science, June 1980, vol. 1, pp 148-156.

Beal, R.C.; Goldfinger, A. D.; Tilly, A. G.; Geckle, W.J. (1981*) System calibration strategies for spaceborne synthetic aperture radar for oceanography.* Report CP 084,Dec.1981. Johns Hopkins University Applied Physics Laboratory.

Frery,A.C.; Müller,H.J.; Yanasse,C.C.; Sant'Anna,J. S. (1997*) A model for extremely heterogeneous clutter.* IEEE Transactions on Geoscience and Remote Sensing, vol. 35, no. 3, pp. 1-12.

Goldfinger, A.D.(1982) *Estimation of spectra from speckled images*. IEEE transactions on aerospace and electronic systems, vol AES-18(5), pp 675-681.

Goodman, J.W (1976) *Some fundamental properties of speckle.* Journal of the optical society of America, Nov. 1976, vol 66, pp 1145-1150.

Jain, A.K. (1989) *Fundamentals of digital image processing*. Prentice Hall International .

Lim, J.S. (1990) *Two-dimensional signal and image processing*. Prentice Hall International .

Tur, M.; Chin, K.C. and Goodman; J.W. (1982) *When is speckle noise multiplicative?.* Appl.

Opt. vol 21, pp 1157-1159.

Yanasse, C.C.F.; Frery, A.C.;Sant'Anna,S.J.S. (1995) *Stochastic distribution and multiplicative model: Relations, properties and aplications to SAR Image Analysis,* Technical Report INPE-5630-NTC/31. INPE, São Jose dos Campos, Brazil, .